Name:

Date:

Start Date of Calc I:

Project 1: Iterative Processes and Chaos

General Directions:

1. Complete the following questions. Questions are written in **bold**. **Type your answers in BOLD using GREEN-colored font next to each question.**
2. Insert photos/tables directly into the document, as directed.
    a. Using a scanner or a free app (like CamScanner) select the option that converts your photos into “Black and White” or “Black and White with Color” **PDF**. This will provide clarity in reading your solutions. Your insertions **should not look like a photo** when submitted.

    b. When using a free PDF converter app, many free apps include a watermark of the software. Make sure that the **watermark does not cover** any of your solutions. If your solution cannot be read, it will not receive credit.

    c. Your file must be **clear and not fuzzy**. Solutions that are not clear will not receive credit.

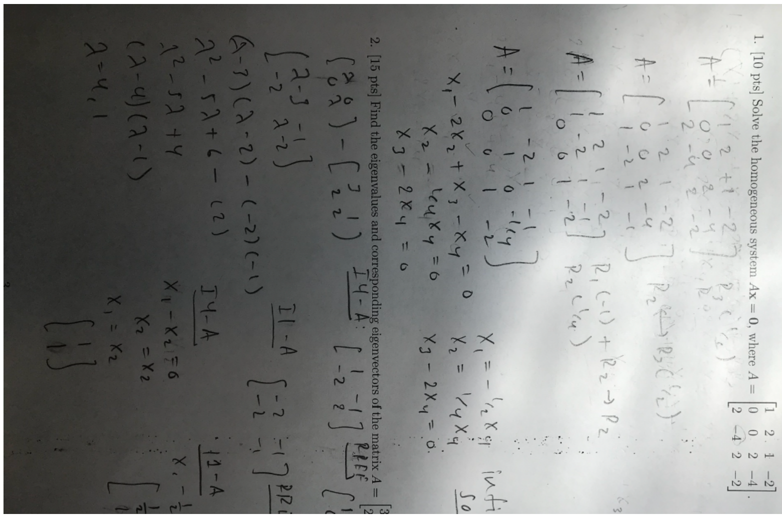
    d. Each of your files should have **your name** on it.

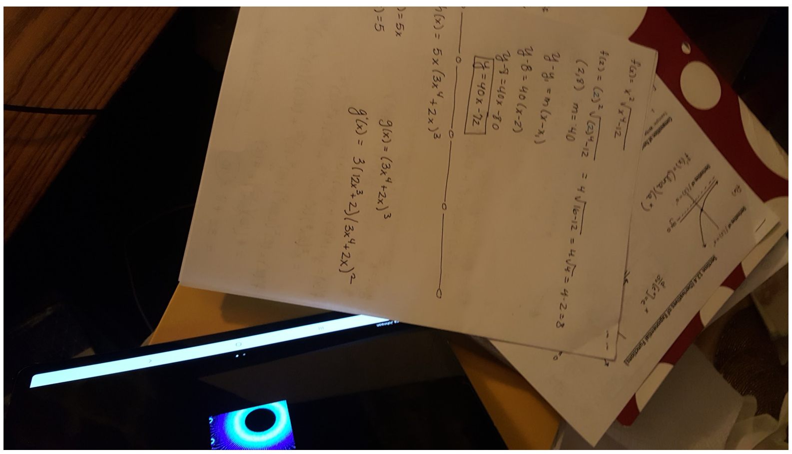
    e. **Crop your insertion** to only include your work. The insertion should not contain other items in view, such as your tabletop or computer.

    f. Insertions should be **orientated facing up** and not upside-down or sideways. The photo should show all of your solutions.

    g. You are expected to clearly communicate your solutions in a **professional and polished manner**. Below is an example of an unacceptable insertion and a properly submitted insertion. Improperly submitted insertions will not receive credit.

        i. Unacceptable Insertions





ii. Proper Insertion

3) $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ K & 0 & 1 \end{bmatrix}$ $KR_1 + R_3 \to R_3$ $\updownarrow$ $(-K)R_1 + R_3 \to R_3$

$\boxed{E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -K & 0 & 1 \end{bmatrix}}$

$E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $R_1 \leftrightarrow R_2$ $\updownarrow$ $R_2 \leftrightarrow R_1$

$\boxed{E_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K \end{bmatrix}$ $KR_3 \to R_3$ $\updownarrow$ $\frac{1}{K} R_3 \to R_3$

$\boxed{E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{K} \end{bmatrix}}$

4) $A = \begin{bmatrix} 4 & -1 \\ 3 & -1 \end{bmatrix}$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

① $R_1 + (-1)R_2 \to R_1$

$\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

② $(-3)R_1 + R_2 \to R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

③ $(-1)R_2 \to R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$I = E_3 E_2 E_1 A$

3. Upload this completed document and any additional files to Blackboard.

Introduction:

Iterative processes can yield some very interesting behavior. In Section 4.9, we saw several examples of iterative processes that converge to a fixed point.

1. **In three to five sentences, describe an iterative process below.**

2. **Describe Newton's method for approximating solutions to f(x)=0 for some function f(x). Write two paragraphs. Insert pictures or diagrams, as necessary.**

3. **Give an example of function f(x) and an initial choice x0 such that Newton's method fails because the iterative process bounces back and forth between two values. This kind of behavior is known as a 2-cycle. Write a paragraph explaining. Insert pictures or diagrams, as necessary.**

Iterative processes can converge to cycles with various periodicities, such as 2-cycles, 4-cycles (where the iterative process repeats a sequence of four values), 8-cycles, and so on. Some iterative processes yield what mathematicians call *chaos*. In this case, the iterative process jumps from value to value in a seemingly random fashion and never converges or settles into a cycle. Although a complete exploration of chaos is beyond the scope of this text, in this project we look at one of the key properties of a chaotic iterative process: sensitive dependence on initial conditions.

4. **In two to three sentences, describe what "sensitive dependence on initial conditions" means. Cite your source.**

Probably the best-known example of chaos is the Mandelbrot set. Perform an internet search. Cite your sources. Answer the following:

5. **Who is the Mandelbrot set named after?**
6. **Where was Mandelbrot born?**
7. **Where was he educated?**
8. **When did Mandelbrot live?**
9. **What was Mandelbrot's profession(s)?**
10. **Mandelbrot spoke about "roughness," "messes," and "chaos." What was his belief about objects that exhibited these qualities?**

11. In three to five sentences, describe the Mandelbrot set.
12. Find a video on YouTube that shows an animated Mandelbrot set. What property of the Mandelbrot set is the animation showing? Insert the link below.
13. How is the Mandelbrot set colored?
14. What sets are closely related to the Mandelbrot set? Find a YouTube video that shows an animation of this set. Insert the link below.

In this project, we will use a logistic map.

15. In a couple of sentences, describe what a logistic map is.

This project will use the logistic map:

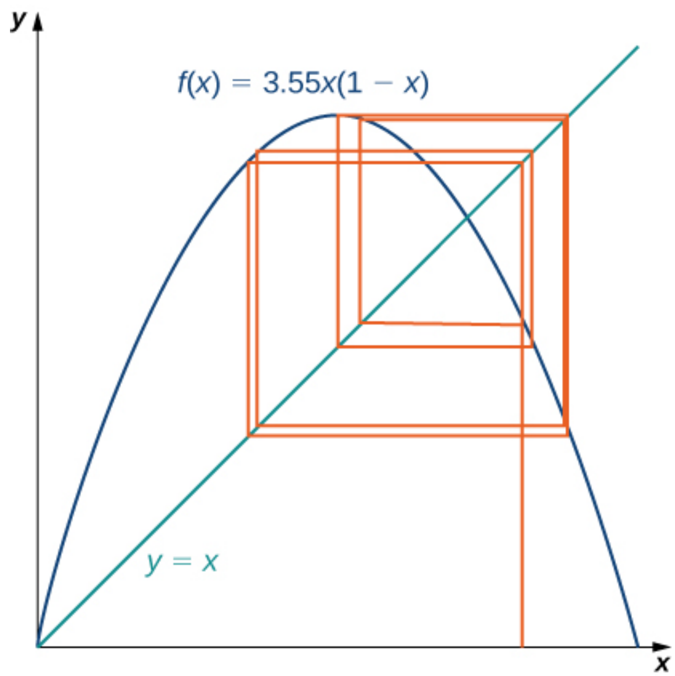
$$F(x) = rx(1-x), \text{ where } x \text{ is an element in } [0, 1] \text{ and } r > 0,$$

is the function in our iterative process. The logistic map is a deceptively simple function; but, depending on the value of *r*, the resulting iterative process displays some very interesting behavior. It can lead to fixed points, cycles, and even chaos.

To visualize the long-term behavior of the iterative process associated with the logistic map, we will use a tool called a *cobweb diagram*. As we did with the iterative process we examined earlier in this section, we first draw a vertical line from the point (x0, 0) to the point (x0, f(x0))=(x0, x1), and continue the process until the long-term behavior of the system becomes apparent.

The figure below shows the long-term behavior of the logistic map when r= 3.55 and x0=0.2. (The first 100 iterations are not plotted.) The long-term behavior of this iterative process is an 8-cycle.

16. Describe how the diagram represents the 8-cycle. (1 to 3 sentences)



17. Let r= 0.5 and choose x0=0.2. Using a spreadsheet (Excel or Google Sheets) that calculates the formula for you, calculate the first 10 values in the sequence. Insert the spreadsheet table below.
    a. State the formula entered into the spreadsheet:
    b. Does the sequence appear to:
        i. Converge? If so, to what value?
        ii. Result in a cycle? If so, what is the period of the cycle?
    c. Insert spreadsheet table.

18. Using a spreadsheet, recalculate when r=2 and x0 remains the same. Calculate the first 10 values in the sequence. Insert the spreadsheet table below. What happens this time?
    a. State the formula entered into the spreadsheet:
    b. Does the sequence appear to:
        i. Converge? If so, to what value?
        ii. Result in a cycle? If so, what is the period of the cycle?
    c. Insert spreadsheet table.

19. For r=3.2 and x0=0.2, calculate the first 49 sequence values. Generate a cobweb design for the iterative process. [First use the Geogebra applet found here]. Insert a screenshot of the cobweb design below. Then generate the cobweb design again using the [MathInsight.org applet]. State the long-term behavior. Insert a screenshot of both graphs generated.

    a. Long-term behavior:
        i. Does it converge? If so, to what value?
        ii. Is it periodic? If so, classify the cycle and state the repeating values.
    b. Insert Geogebra cobweb design screenshot.
    c. Insert MathInsight.org graphs.

20. **For r=3.5 and x0=0.2, calculate the first 49 sequence values. Generate a cobweb design for the iterative process. [First use the Geogebra applet found here]. Insert a screenshot of the cobweb design below. Then generate the cobweb design again using the [MathInsight.org applet]. State the long-term behavior. Insert a screenshot of both graphs generated.**

    a. **Long-term behavior:**
        i. **Does it converge? If so, to what value?**
        ii. **Is it periodic? If so, classify the cycle and state the repeating values.**
    b. **Insert Geogebra cobweb design screenshot.**
    c. **Insert MathInsight.org graphs.**

21. **For r=4 and x_0=0.2, calculate the first 49 sequence values. Generate a cobweb design for the iterative process. [First use the Geogebra applet found here]. Insert a screenshot of the cobweb design below. Then generate the cobweb design again using the [MathInsight.org applet.] State the long-term behavior. Insert a screenshot of both graphs generated.**

    a. **Long-term behavior:**
        i. **Does it converge? If so, to what value?**
        ii. **Is it periodic? If so, classify the cycle and state the repeating values.**
    b. **Insert Geogebra cobweb design screenshot.**
    c. **Insert MathInsight.org graphs.**

22. **Repeat the process for r=4, but modify x0 slightly to x0=0.201. Calculate the first 49 sequence values. Generate a cobweb design for the iterative process. [First use the Geogebra applet found here]. Insert a screenshot of the cobweb design below. Then generate the cobweb design again using the [MathInsight.org applet]. State the long-term behavior. Insert a screenshot of both graphs generated.**

    a. **Long-term behavior:**
        i. **Does it converge? If so, to what value?**
        ii. **Is it periodic? If so, classify the cycle and state the repeating values.**
    b. **Our new initial guess is only 0.001 away from our previous initial guess of 0.2. How do the function values compare between these two different initial guesses?**
    c. **Insert Geogebra cobweb design screenshot.**
    d. **Insert MathInsight.org graphs.**

23. **So what is going on with these processes? How is the Mandelbrot set related to a logistic map? Can order arise from chaos, and where do we see it? Watch the following video to see.**

    - ***[This Equation Will Change How You See the World (the Logistic Map)]*** **(18:38)**

      Citation: Veritasium. (2020, January 29). *This equation will change how you see the world (the logistic map)* [Video]. YouTube. https://www.youtube.com/watch?v=ovJcsL7vyrk&feature=emb_rel_end

24. Video presentation: Prepare a video explaining your solutions to #19, 20, 21. Your video must include the following (see video assignment directions video):
    a. An introduction name portion. You will state your name and hold up a piece of paper with your printed name, signature, and date.
    b. A time stamp from the calendar on your computer.
    c. Presentation explaining your process and mathematical steps.

25. Questions 1–24 are required components to achieve a C grade (max 79%) on this project. To achieve an A grade (max 100%), ONE of the following (a or b) must also be completed.

    a. Answer the following question on the Iterative Process and Chaos Solution Sheet. (Submit a 5- to 10-minute video with your solution. The video must present your complete epsilon delta proof.) Your video must also include the following:
        i. An introduction name portion. You will state your name and hold up a piece of paper with your printed name, signature, and date.
        ii. A time stamp from the calendar on your computer.
        iii. Video presentation explaining your process and mathematical steps.

    Question: For r=1.5 and x0=0.2
        1. Does this sequence converge?
        2. Does a limit exist for this sequence? If so, what is the notation?
        3. Suppose the function defined for the logistic map above did not define an iterative process but simply a function (like we have considered thus far in the course.) Provide a complete epsilon delta proof establishing this limit.

    b. Oral presentation and defense of your project via Zoom with instructor (30 minutes): During this time, your instructor will ask you questions regarding your process and calculations in your submitted project. You may also be asked to perform new calculations and interpret the results during the presentation.